

# Temperature dependence of dimension 6 gluon operators and their effects on Charmonium

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Starting from an earlier representation of the independent dimension 6 gluon operators in terms of color electric and magnetic fields, we estimate their changes near the critical temperature  $T_c$  using the temperature dependence of the dimension 4 electric and magnetic condensates extracted from pure gauge theory on the lattice. We then improve the previous QCD sum rules for the  $J/\psi$  mass near  $T_c$  based on dimension 4 operators, by including the contribution of the dimension 6 operators to the OPE. We find an enhanced stability in the sum rule and confirm that the  $J/\psi$  will undergo an abrupt change in the property across  $T_c$ .

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## I. INTRODUCTION

The temperature dependence of the gluon condensate, or the trace anomaly in the pure gauge theory, offers a useful picture on the nature of the QCD phase transition [1]. The scalar gluon condensate together with the twist-2 gluon operator are the two independent gluon operators at dimension 4. These operators can be re-expressed in terms of the electric and the magnetic condensate. The temperature dependence of these operators can be calculated directly from lattice calculations of the space-time and space-space elementary plaquette [1, 2], or from combining the calculation of the energy density and pressure [3]. The calculations show that while there is a rapid change of the electric condensate across the phase transition temperature, the magnetic condensate changes very little [3], somehow reflecting a possible link to the observed sudden change in the space-time Wilson loop and the persistent area-law behavior of the space-space Wilson loop across the phase transition [4].

Using the temperature dependence of the dimension 4 condensates as the input in the QCD sum rule approach for the heavy quark system,  $J/\psi$  and  $\eta_c$  have been found to undergo a rapid property change across the phase transition [3, 5–7] and to their dissociation [8, 9] slightly above the critical temperature.

In a recent work, two of us (H.K. and S.H.L.) have performed the renormalization of all the independent dimension 6 spin-2 gluon operators and found the scale invariant combination in the pure gauge theory [10]. Together with the dimension 6 scalar condensate, whose renormalization has been worked out before [11, 12], the renormalization of the two dimension 6 gluon operators are now understood.

Unfortunately, a direct lattice calculation is presently not feasible to extract their temperature dependence since the higher dimensional operators will come with large uncertainties and the mixing with lower dimensional operators becomes problematic. At the same time, in the pure gauge theory, it is interesting to note that

while the dimension 6 scalar operator is composed of a higher product of gluon field strength tensor, the spin-2 part is the second moment of the dimension 4 scalar gluon condensate, thus suggesting a strong correlation with the dimension 4 operator. The connections become more apparent when we express the dimension 6 operators in terms of color electric ( $E$ ) and magnetic ( $B$ ) fields.

In this paper, using these expressions and the temperature dependence of dimension 4 electric and magnetic condensates extracted from lattice gauge theory, we estimate the changes of the dimension 6 operators near the critical temperature  $T_c$ . Furthermore, we then improve the previous QCD sum rules for  $J/\psi$  mass near  $T_c$ , which was based on dimension 4 operators, by including the contribution of the dimension 6 operators to the OPE, whose nuclear medium effect was previously studied in Ref. [13]. We find an enhanced stability in the sum rule and confirm that the  $J/\psi$  will undergo an abrupt change in the property across  $T_c$  [5, 6].

In section II, we will show  $E$  and  $B$  fields representations of dimension 6 gluon operators. In section III, we will show the temperature dependence of the condensates. In section IV, we will apply our condensates to the sum rules. Section V is devoted to a summary.

## II. FIELD REPRESENTATION

At dimension 4, there are two independent gluon operators that can be constructed from  $G_{\mu\alpha}^a G_{\nu\alpha}^a$ . These are the scalar and the twist-2 gluon operators.

$$\begin{aligned} g_{\mu\nu}[G_{\mu\alpha}^a G_{\nu\alpha}^a] &= G_{\mu\nu}^a G_{\mu\nu}^a \\ G_{\mu\alpha}^a G_{\nu\alpha}^a|_{ST} &= (u_\mu u_\nu - g_{\mu\nu}/4)G_2, \end{aligned} \quad (1)$$

where the subscript  $ST$  represents symmetric and traceless indices. These operators can be represented by  $E$

and  $B$  fields as follows.

$$\begin{aligned} G_{\mu\nu}^a G_{\mu\nu}^a &= 2(B^2 - E^2) \\ G_2 &= -\frac{2}{3}(E^2 + B^2), \end{aligned} \quad (2)$$

where the trace is taken for the color indices of  $E$  and  $B$  fields and the medium four vector is taken to be  $u^\mu = (1, 0, 0, 0)$  in this work.

For dimension 6 operators, in addition to the twist-2 gluon operator, there are two more independent dimension 6 gluon operators that remain after using the equations of motion in the pure gauge theory. Introducing the short hand notation  $G_{\mu\nu}^3 \equiv f^{abc} G_{\mu\alpha}^a G_{\alpha\beta}^b G_{\beta\nu}^c$ , one finds these operators in a similar form to the dimension 4 case,

$$\begin{aligned} g_{\mu\nu}[G_{\mu\nu}^3] &= f^{abc} G_{\mu\alpha}^a G_{\alpha\beta}^b G_{\beta\nu}^c \\ G_{\mu\nu}^3|_{ST} &= (u_\mu u_\nu - g_{\mu\nu}/4)G_3. \end{aligned} \quad (3)$$

These operators can also be represented by the  $E$  and  $B$  fields. Using parity and rotational symmetry, we find that  $E^a E^b E^c$  and  $B^a B^b E^c$  type of operators vanish such that the remaining forms are given as follows:

$$\begin{aligned} g_{\mu\nu}[G_{\mu\nu}^3] &= f^{abc}[3B^a \cdot (E^b \times E^c) - B^a \cdot (B^b \times B^c)] \\ G_3 &= \frac{f^{abc}}{3} B^a \cdot (B^b \times B^c + E^b \times E^c). \end{aligned} \quad (4)$$

In the following we will abbreviate the triple scalar product of fields  $f^{abc} A^a \cdot (B^b \times C^c)$  as  $ABC$  for simplicity. Using this notation, the first and second line of Eq. (4) will be represented as  $[3BEE - BBB]$  and  $[BBB + BEE]/3$  respectively.

### III. TEMPERATURE DEPENDENCE OF CONDENSATES

We start from the temperature dependence of  $\langle \frac{\alpha_s}{\pi} E^2 \rangle_T$  and  $\langle \frac{\alpha_s}{\pi} B^2 \rangle_T$  extracted from lattice calculations [3] and discuss how the temperature dependence of higher dimensional operators can be estimated.

The temperature dependent dimension 4 condensates can be expressed as follows.

$$\left\langle \frac{\alpha_s}{\pi} G_{\mu\alpha}^a G_{\mu\alpha}^a \right\rangle_T = 2 \left[ \left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_T - \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_T \right] \quad (5)$$

$$\left\langle \frac{\alpha_s}{\pi} G_2 \right\rangle_T = -\frac{2}{3} \left[ \left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_T + \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_T \right]. \quad (6)$$

For the dimension 6 condensates, we first note that all dimension 6 operators with either spin 0 or 2 can be represented as linear combinations of the operators in Eq. (3) in pure gauge theory [13], so that we can represent the operators with two covariant derivatives as

$$\begin{aligned} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a G_{\mu\nu;\kappa\kappa}^a \right\rangle_T &= 2 \left\langle \frac{g\alpha_s}{\pi} G_{\mu\nu}^3 \right\rangle_T \\ &= \frac{4}{\pi^{1/2}} [3\langle \alpha_s^{3/2} BEE \rangle_T - \langle \alpha_s^{3/2} BBB \rangle_T], \end{aligned} \quad (7)$$

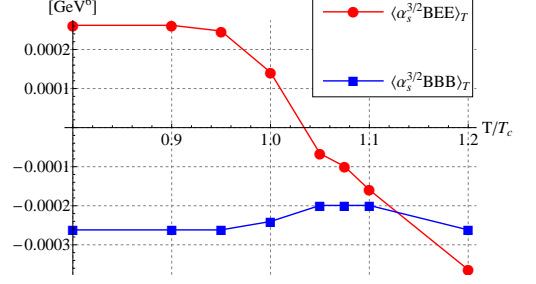


FIG. 1: Temperature dependence of BBB and BEE.

$$\begin{aligned} X_T &= 2 \left\langle \frac{g\alpha_s}{\pi} G_3 \right\rangle_T \\ &= \frac{4}{3\pi^{1/2}} [\langle \alpha_s^{3/2} BEE \rangle_T + \langle \alpha_s^{3/2} BBB \rangle_T], \end{aligned} \quad (8)$$

where we have defined

$$\left\langle \frac{\alpha_s}{\pi} G_{\kappa\lambda}^a G_{\kappa\lambda;\mu\nu}^a \right\rangle_T = \left( u_\mu u_\nu - \frac{1}{4} g_{\mu\nu} \right) X_T. \quad (9)$$

To estimate the temperature dependencies of  $BEE$  and  $BBB$  from temperature dependencies of the lower dimensional field operators  $E^2$  and  $B^2$ , we assume that fields are isotropic and that the angular correlations can be neglected. Namely,  $|E_i^a| = E$  or 0 and  $|B_i^a| = B$  or 0 for  $a = 1, \dots, 8$ . Such an assumption is actually satisfied in an instanton configuration where  $E_i^a, B_i^a \propto \delta_{ai}$  [15, 16] and is used successfully in the vacuum [17].<sup>1</sup>

Hence, we approximate

$$\langle \alpha_s^{3/2} BEE \rangle_T = \langle \alpha_s^{3/2} BEE \rangle_0 \frac{\langle \frac{\alpha_s}{\pi} B^2 \rangle_T^{1/2} \langle \frac{\alpha_s}{\pi} E^2 \rangle_T}{\langle \frac{\alpha_s}{\pi} B^2 \rangle_0^{1/2} \langle \frac{\alpha_s}{\pi} E^2 \rangle_0} \quad (10)$$

$$\langle \alpha_s^{3/2} BBB \rangle_T = \langle \alpha_s^{3/2} BBB \rangle_0 \frac{\langle \frac{\alpha_s}{\pi} B^2 \rangle_T^{3/2}}{\langle \frac{\alpha_s}{\pi} B^2 \rangle_0^{3/2}}. \quad (11)$$

Furthermore, using the fact that  $fG^3 = 4BEE = -4BBB$  in the Euclidean spacetime at zero temperature and approximating  $\langle g^3 fG^3 \rangle_0 = (0.6 \text{ GeV})^6$  [17], we can determine the vacuum values at zero temperature.

$$\langle \alpha_s^{3/2} BEE \rangle_0 = -\langle \alpha_s^{3/2} BBB \rangle_0 = \frac{(0.6 \text{ GeV})^6}{4(4\pi)^{3/2}}. \quad (12)$$

The temperature dependencies of Eqs. (10) and (11) are then given in Fig. 1. Furthermore, the temperature dependence of the operators in Eq. (7) and Eq. (8) are accordingly obtained.

<sup>1</sup> It is not trivial that such a configuration still holds at high temperature. However, since it gives the maximum of the triple scalar product, our estimate can be regarded as conservative upper bound.

#### IV. APPLICATION TO SUM RULES FOR $J/\psi$

Using the temperature dependence of the dimension 4 gluon operators, two of us have calculated the mass of  $J/\psi$  using QCD moment sum rules near the critical temperature [5, 6]. There, it was found that the properties of the  $J/\psi$  underwent a sudden change at  $T_c$ . However, the sum rule was also found to become unstable above  $1.05T_c$ . This instability was later found to be linked to an onset of the broadening [7] and to a precursor effect of the melting of  $J/\psi$  at slightly higher temperature as being found by application of the Maximum Entropy method to the sum rule [8]. To confirm the QCD sum rule analysis with better stability, we generalize the sum rule analysis to include the contributions from dimension 6 operators.

For that purpose, we have to further include the temperature dependence of dimension 6 twist-2 operator, which we have not discussed so far. This operator can not be expressed in terms of  $E$  and  $B$  fields directly and is parametrized as follows:

$$\begin{aligned} & \left\langle \frac{\alpha_s}{\pi} G_{\mu\kappa}^a G_{\nu\kappa;\alpha\beta}^a |_{ST} \right\rangle \\ &= G_4 [u_\mu u_\nu u_\alpha u_\beta + \frac{1}{48} (g_{\mu\nu} g_{\alpha\beta} + g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \\ & - \frac{1}{8} (u_\mu u_\nu g_{\alpha\beta} + u_\mu u_\alpha g_{\nu\beta} + u_\mu u_\beta g_{\alpha\nu} + u_\nu u_\alpha g_{\mu\beta} \\ & + u_\nu u_\beta g_{\mu\alpha} + u_\alpha u_\beta g_{\mu\nu})]. \end{aligned} \quad (13)$$

It should be noted that  $G_4$  is the higher moment of the dimension 4 twist-2  $G_2$  operator in Eq. (1). To estimate the value of  $G_4$  we use the estimate near  $T_c$  based on a quasiparticle picture given in Ref.[14]. There, the ratio between  $G_4$  and  $G_2$  is given by the square of the mass of the quasiparticle times  $A_4/A_2$ , where  $A_i$  is the  $i$ -th moment of the structure function of the quasiparticle. Hence we will assume

$$G_4/G_2 \sim -m_G^2 A_4/A_2. \quad (14)$$

For the numerical value, we take  $m_G = 0.6$  GeV for the thermal gluon mass near  $T_c$  and take  $A_4 = 0.02$  and  $A_2 = 0.9$  for a typical value for any hadron as discussed in given in Ref. [13]; such approximation should be valid below  $T_c$ . To extend the formula to temperature above  $T_c$ , we make use of the thermal fluctuation of the thermal gluon momentum assuming a temperature dependent thermal mass as extracted from lattice gauge theory [18]. Noting that  $G_4$  involves two additional covariant derivatives in comparison to  $G_2$ , we will use the following approximation.

$$G_4/G_2 \sim - \left( m_G^2 \frac{A_4}{A_2} \right) \frac{\langle p^2 \rangle_T}{\langle p^2 \rangle_{T_c}}, \quad (15)$$

where we take

$$\frac{\langle p^2 \rangle_T}{\langle p^2 \rangle_{T_c}} = \frac{\int_0^\infty dp p^4 n_B(m_{\text{eff}}(T), T)}{\int_0^\infty dp p^4 n_B(m_{\text{eff}}(T_c), T_c)} \Big|_{T_c=260\text{MeV}} \quad (16)$$

with  $n_B(m, T)$  being the Bose distribution function  $(e^{\sqrt{p^2+m^2}/T} - 1)^{-1}$  for gluons and

$$m_{\text{eff}}(T) = 0.6 + 0.062 \left( \frac{T}{T_c} - 1.5 \right)^2 \text{ GeV}, \quad (17)$$

as parametrized from Ref. [18].

We now calculate the operator product expansion (OPE) of the following correlation function of the vector current  $j_\mu = \bar{c}\gamma_\mu c$ .

$$\Pi(q^2) = \frac{-1}{3q^2} \int d^4x e^{iqx} \langle T[j_\mu(x), j^\mu(0)] \rangle, \quad (18)$$

Taking the moments

$$M_n(Q_0^2) = \frac{1}{n!} \left( -\frac{d}{dQ^2} \right)^n \Pi(Q^2)|_{Q^2=Q_0^2}, \quad (19)$$

we find the following form up to dimension 4 operators [19]

$$M_n(\xi) = A_n^V(\xi) [1 + a_n(\xi)\alpha_s + b_n(\xi)\phi_b^4 + c_n(\xi)\phi_c^4] \quad (20)$$

$$\phi_b^4 = \frac{4\pi^2}{9} \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{(4m_c^2)^2} \quad (21)$$

$$\phi_c^4 = \frac{4\pi^2}{3} \frac{G_2}{(4m_c^2)^2}. \quad (22)$$

The additions from dimension 6 operators are of the following form [13].

$$\Delta M_n^6(\xi) = A_n^V(\xi) [s_n(\xi)\phi_s^6 + x_n(\xi)\phi_x^6 + g_{4n}(\xi)\phi_{g_4}^6] \quad (23)$$

$$\phi_s^6 = \frac{4\pi^2}{3 \cdot 1080} \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu;\kappa\kappa}^a \rangle}{(4m_c^2)^3} \quad (24)$$

$$\phi_x^6 = \frac{9}{2} \frac{4\pi^2}{3 \cdot 1080} \frac{X}{(4m_c^2)^3} \quad (25)$$

$$\phi_{g_4}^6 = 10 \frac{4\pi^2}{3 \cdot 1080} \frac{G_4}{(4m_c^2)^3}, \quad (26)$$

where the Wilson coefficients are summarized in Ref. [13] and we take  $\xi = Q_0^2/(4m_c^2) = 1$ .

Before calculating the mass of  $J/\psi$ , it is useful to discuss the effects of adding each contribution from dimension 4 and 6 condensates. The left and right graphs of Fig. 2 show the total contributions to moments  $M_n(\xi)$  from dimension 4 and 6 condensates, respectively, divided by the perturbative contribution at  $T = 0.8, 1.05, 1.2T_c$ ; the result at  $T = 0.8T_c$  being almost identical to that at  $T = 0$ . One notes that at  $T = 1.05T_c$ , the contributions change signs in both figures. This suggests that the finite temperature corrections become slightly larger than the vacuum condensate value, which shows the onset of large temperature correction. The correction at  $T = 1.2T_c$  are twice as large as

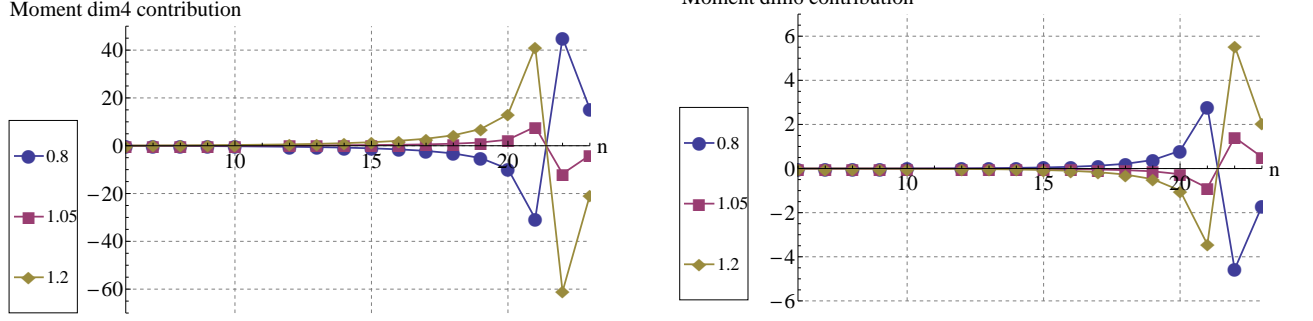


FIG. 2: The left and right shows the contributions to  $M_n(\xi)$  from dimension 4 and dimension 6 condensates, respectively, divided by perturbative contribution. The circle, square and diamond represent  $T = 0.8, 1.05$  and  $1.2T_c$  respectively.

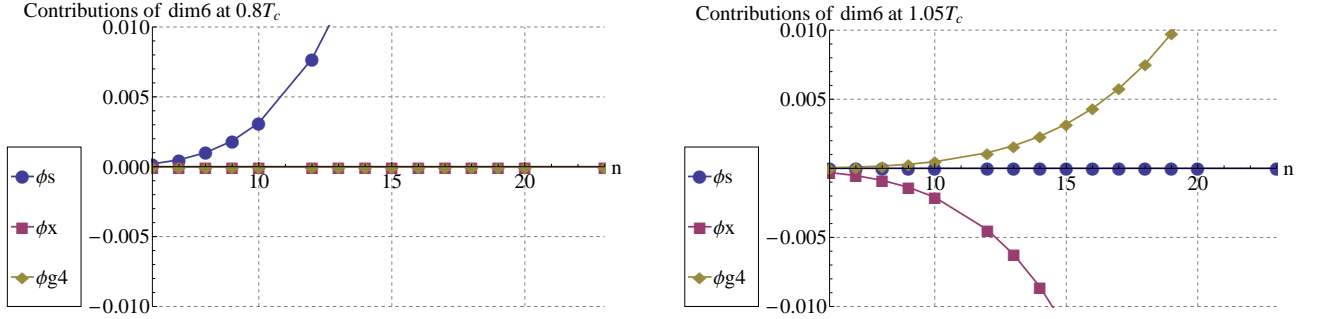


FIG. 3: Contributions to  $M_n(\xi)/A_n^V(\xi)$  of dimension 6 condensates at  $0.8T_c$  (left figure) and at  $1.05T_c$  (right figure).

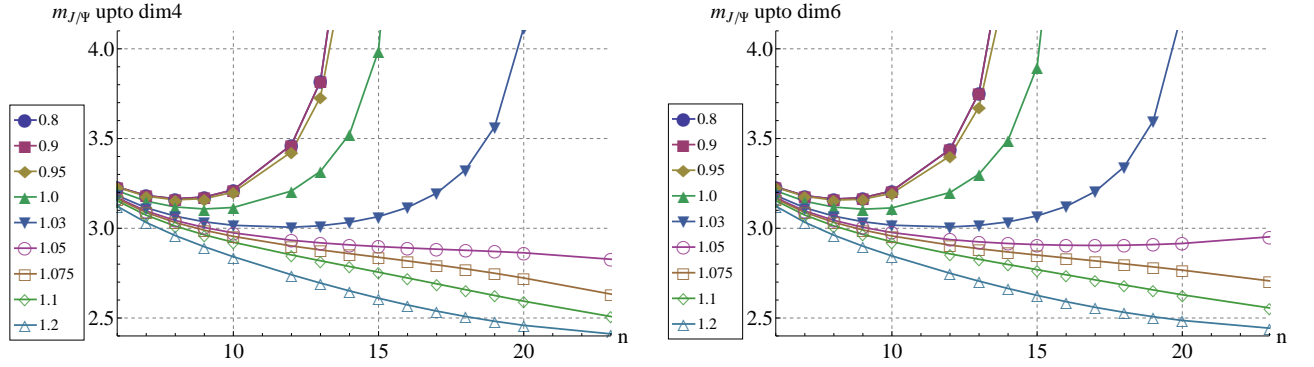


FIG. 4: Temperature dependence of  $m_{J/\psi}$ . Left: considering up to dim4 condensates. Right: up to dim 6 condensates.

the vacuum value with opposite sign suggesting the need to consider improved method to consider temperature effects. Furthermore, for each temperature, the moments change sign after  $n = 21$ . As higher  $n$  are more sensitive to higher orders of power correction, this signals the onset of the breakdown of the OPE.

Fig. 3 shows the contributions to moments  $M_n(\xi)/A_n^V(\xi)$  of each dimension 6 condensates at  $T = 0.8$  (left figure) and  $1.05T_c$  (right figure). As can be seen from the left figure, the scalar condensate dominates for any  $n$  at  $T = 0.8T_c$ . On the other hand, the right figure shows that the temperature dependent  $\phi_x$  and  $\phi_{g4}$

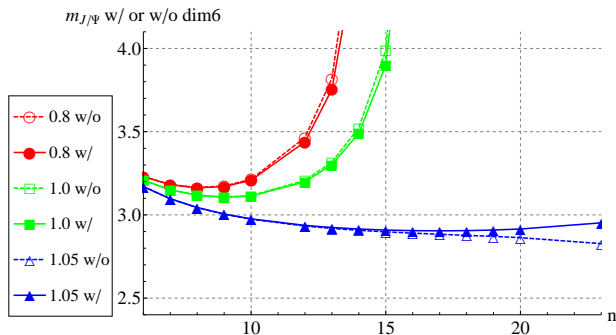


FIG. 5:  $m_{J/\psi}$  at  $T = 0.8, 1.0, 1.05T_c$  with or without dimension 6 condensates

dominates over the scalar condensate. Moreover, while the absolute value of  $\phi_{g4}$  is larger than  $\phi_x$ , they have opposite signs and tend to cancel the contributions to the moment each other, as also seen from the right of Fig. 2. Again this signals the onset of large temperature dependence at  $T = 1.05T_c$ .

Let us now look at the mass of  $J/\psi$ . Assuming that the imaginary part of the correlation function is dominated by the lowest pole, the mass is obtained from

$$m_{J/\psi}^2 = \frac{M_{n-1}}{M_n} - 4m_c^2. \quad (27)$$

Now, we adopt the temperature dependent condensate values as discussed in the previous section to calculate the temperature dependent mass.

Fig. 4 shows the moment sum rule for the mass as given in Eq. (27). The left panel of Fig. 4 shows the result when only the contributions from dimension 4 operators are considered. The right panel shows the result

after contributions from dimension 6 operators are added. One notes that while the values of the mass shift do not change much, the stability of the sum rules with dimension 6 condensate improves over the sum rules with dimension 4 condensate. Specifically, the instability in the sum rule with dimension 4 condensate at  $1.05T_c$  turns into a stable plateau structure by including dimension 6 condensate, as can be seen in Fig. 5. The stability in  $n$  guarantees that the assumptions of the operator product expansion side and phenomenological side are both valid. Therefore, including the contribution from dimension 6 operators seems to extend the region of stability to slightly higher temperature. This suggest that the  $J/\psi$  will still survive to this temperature.

## V. SUMMARY

We have introduced a parametrization of the temperature dependence of the dimension 6 gluon operators based on the temperature dependence of the dimension 4 electric and magnetic condensates extracted from lattice gauge theory. We then improved the previous QCD sum rules for the  $J/\psi$  mass near  $T_c$  based on dimension 4 operators, by including the contribution of the temperature dependent dimension 6 operators to the OPE. We find that the addition extends the stability in the sum rule up to slightly higher temperature of  $1.05T_c$ .

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- [1] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, Nucl. Phys. B **469**, 419 (1996).
  - [2] S. H. Lee, Phys. Rev. D **40**, 2484 (1989).
  - [3] S. H. Lee and K. Morita, Phys. Rev. D **79**, 011501 (2009).
  - [4] E. Manousakis and J. Polonyi, Phys. Rev. Lett. **58**, 847 (1987).
  - [5] K. Morita and S. H. Lee, Phys. Rev. Lett. **100**, 022301 (2008).
  - [6] K. Morita and S. H. Lee, Phys. Rev. C **77**, 064904 (2008).
  - [7] K. Morita and S. H. Lee, Phys. Rev. D **82**, 054008 (2010).
  - [8] P. Gubler, K. Morita and M. Oka, Phys. Rev. Lett. **107**, 092003 (2011).
  - [9] C.A. Dominguez, M. Loewe, J.C. Rojas and Y. Zhang Phys. Rev. D **81**, 014007 (2010); **83** 034033 (2011).
  - [10] H. Kim and S. H. Lee, Phys. Lett. B **748**, 352 (2015).
  - [11] S. Narison and R. Tarrach, Phys. Lett. B **125**, 217 (1983).
  - [12] R. Tarrach, Nucl. Phys. B **196**, 45 (1982).
  - [13] S. H. Lee and S. S. Kim, Nucl. Phys. A **679**, 517 (2001).
  - [14] K. Morita and S. H. Lee, Phys. Rev. C **85**, 044917 (2012).
  - [15] A. A. Belavin, A. M. Polyakov, A. S. Schwartz and Y. S. Tyupkin, Phys. Lett. B **59**, 85 (1975).
  - [16] T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
  - [17] S. N. Nikolaev and A. V. Radyushkin, Nucl. Phys. B **213**, 285 (1983).
  - [18] P. Levai and U. W. Heinz, Phys. Rev. C **57**, 1879 (1998).
  - [19] F. Klingl, S. s. Kim, S. H. Lee, P. Morath and W. Weise, Phys. Rev. Lett. **82**, 3396 (1999); [Phys. Rev. Lett. **83**, 4224 (1999)].